TRANSFER FUNCTION REPRESENTATION

1- Block Diagrams

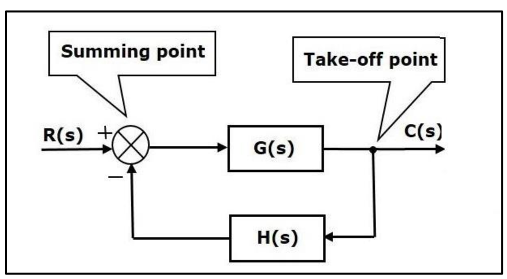
Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form. In order to draw the block diagram of a practical system each element of practical system is represented by a block. For a closed loop system, the function of comparing the different signals is indicated by the summing point while a point from which signal is taken for the feedback purpose is indicated by take off point in block diagrams.

A block diagram has following five elements associated with it.

1) Functional Blocks
2) Transfer functions of elements shown inside the functional blocks
3) Summing points
4) Take off points
5) Arro

1-1- Basic Elements of Block Diagram.

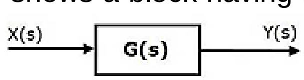
The basic elements of a block diagram are a blocks, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.



The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

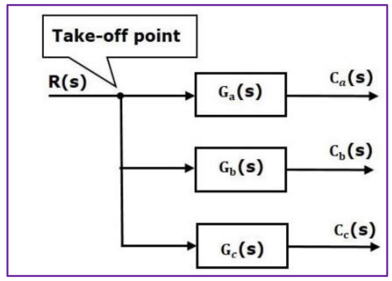
1- ***Block***

The transfer function of a component is represented by a block. Block has single input and single output. The following figure shows a block having input X(s), output Y(s) and the transfer function G(s)

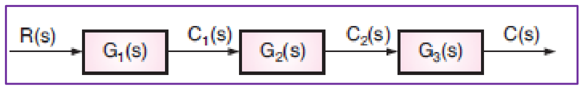


$$= \frac{Y(S)}{X(S)} = G(S)$$

2- Take off point. Application of one input source to two or more systems is represented by a takeoff point as shown at point *A* in Fig. 3.1.2. It should be noted that such taking off from the input signal does not alter the input signal.



3- Blocks in *Series*. also called cascade connection. When several blocks are connected in cascade the overall equivalent transfer function is determined below :

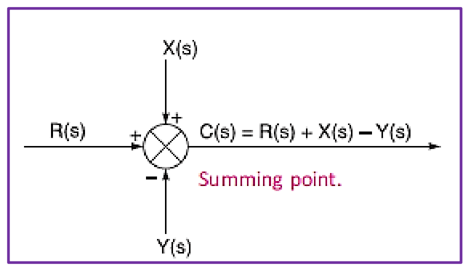


$$\frac{C_1(s)}{R(s)}\cdot\frac{C_2(s)}{C_1(s)}\cdot\frac{C(s)}{C_2(s)} = G_1(s)\cdot G_2(s)\cdot G_3(s)$$

Therefore, the overall equivalent transfer function $\frac{C(s)}{R(s)} = G_1(s)\cdot G_2(s)\cdot G_3(s)$

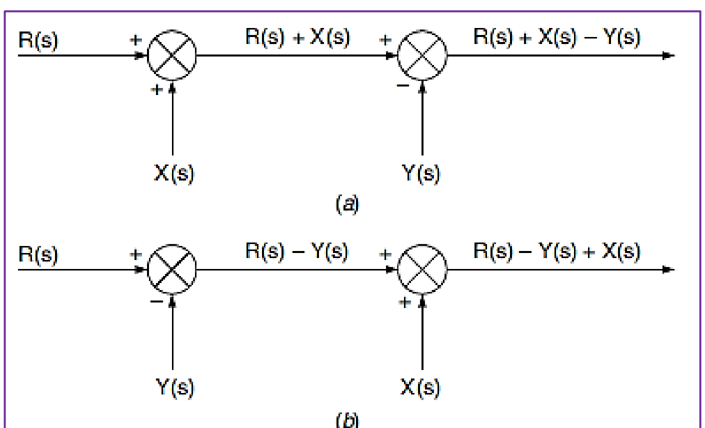
The equivalence of blocks is *Series* is shown by Fig.

4- Summing point. Summing point represents summation of two or more input signals entering in a system. The output of a summing point being the sum of the entering inputs. Summing point is represented in Fig. beside It is necessary to indicate the sign specifying the input signal entering a summing point.



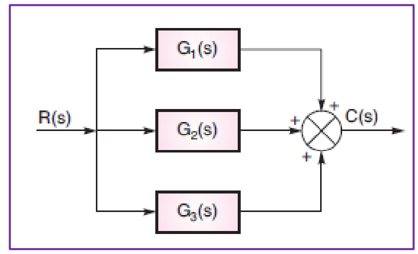
Summing point.

- Consecutive summing points can be interchanged, as this interchange does not alter the output signal as shown in Fig. below *(a)* and *(b).*



Interchange of consecutive summing points.

5- **Blocks in parallel.** The blocks which are connected in **parallel** will have the **same input**. In the following figure, three blocks having transfer functions $G_1(s)G_1(s)$ $G_3(s)$ and) are connected in parallel. The outputs of these three blocks are connected to the summing point .



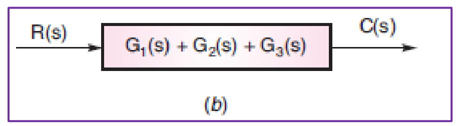
The overall equivalent transfer function is determined below,

$$C(s) = R(s)\,G_1(s) + R(s)\,G_2(s) + R(s)\,G_3(s)$$
$$C(s) = R(s)\,[G_1(s) + G_2(s) + G_3(s)]$$

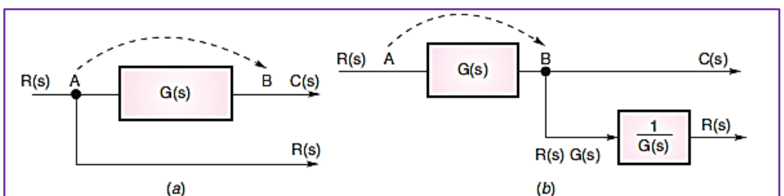
Therefore, the overall equivalent transfer function is,

$$\frac{C(s)}{R(s)} = [G_1(s) + G_2(s) + G_3(s)]$$

The equivalence of Fig. above is shown by Fig. below .

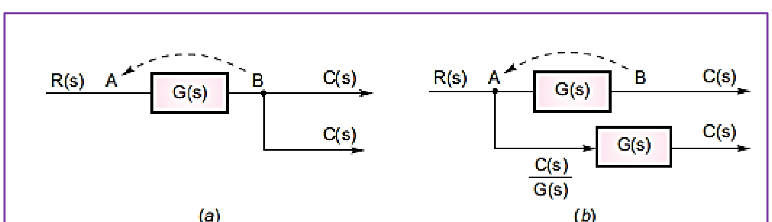


6- Shifting of a take off point from a position before a block to a position after the block is shown in Fig. *(a)* and *(b).*



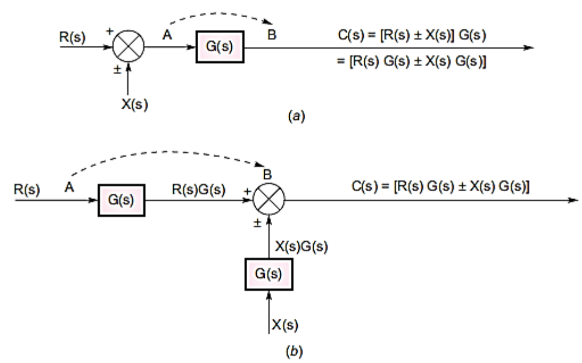
Take off point at position *A* shifted to position *B*.

7- Shifting of a take off point from a position after a block to a position before the block is shown in Fig. *(a)* and *(b).*



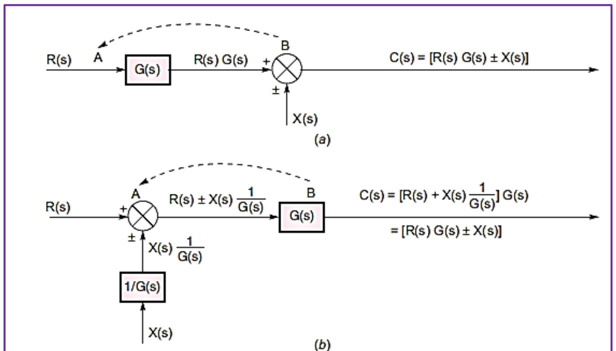
Take off point at position *B* shifted to position *A*.

8- Shifting of a summing point from a position before a block to a position after the block is shown in Fig. *(a)* and *(b).*



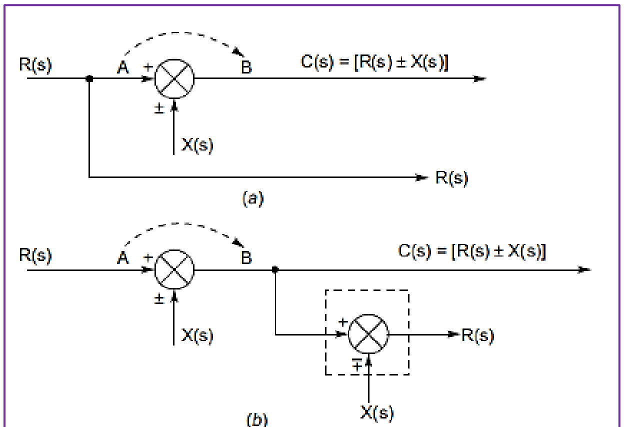
Summing at position *A* is shifted to position *B*.

9- Shifting of a summing point from a position after a block to a position before the block is shown in Fig. *(a)* and *(b).*



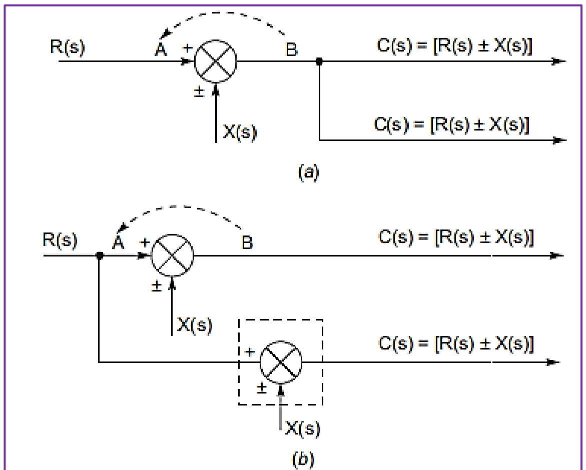
Summing point at position *B* shifted to position *A*.

10- Shifting of a takeoff point from a position before a summing point to a position after the summing point is shown in Fig. *(a)* and *(b).*



Take off point at position *A* is shifted to position *B*.

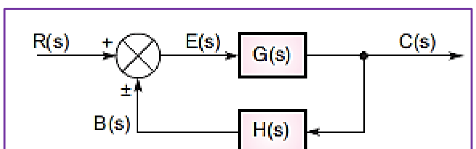
11- Shifting of a take off point from a position after a summing point to a position before the summing point is shown **in** *(a)* and *(b).*



Take off point at position *B* shifted to position *A*.

12- Feedback Connection

As we discussed in previous chapters, there are two types of feedback-positive feedback and negative-feedback. The following figure shows negative -feedback or positive- feedback control system. Here, two blocks having transfer functions G(s) and H(s) form a closed-loop control system is represented by a block diagram shown in Figure below



Block diagram of a closed-loop control system.

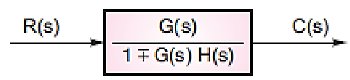
wherein a fraction of the output *B(s)* = C(s) * *H(s)* is compared with the input *R(s)* which results in an error *E(s)* given by

$$E(s) = [R(s) \pm B(s)]$$

From block diagram C(s)/E(s)=G(s) and B(s)/C(s)=H(s) .sub .in equation above ,we get C(s)/G(s)=R(s) ±C(s)H(s), then R(s)=C(s)[1/G(s) ±H(s)], Simplifying, we get overall transfer function denoted as *T(s)* is given by

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{(1 \pm G(s)H(s)}$$

The equivalent transfer function is represented by a single block diagram shown in Fig. below .



1-5 Block Diagram Reduction Rules

Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

Rule 1 − Check for the blocks connected in series and simplify.

Rule 2 − Check for the blocks connected in parallel and simplify.

Rule 3 − Check for the blocks connected in feedback loop and simplify.

Rule 4 − If there is difficulty with take-off point while simplifying, shift it towards right.

Rule 5 − If there is difficulty with summing point while simplifying, shift it towards left.

Rule 6 − Repeat the above steps till you get the simplified form, i.e., single block.

Note –

1- The transfer function present in this single block is the transfer function of the overall block diagram.

2- Follow these steps in order to calculate the transfer function of the block diagram having multiple inputs.

Step 1 − Find the transfer function of block diagram by considering one input at a time and make the remaining inputs as zero.

Step 2 − Repeat step 1 for remaining inputs.

Step 3 − Get the overall transfer function by adding all those transfer functions.

SOLVED EXAMPLES

Example 1. *Determine the transfer function C(s)/ R(s) from the block diagram shown in Fig. 1.*

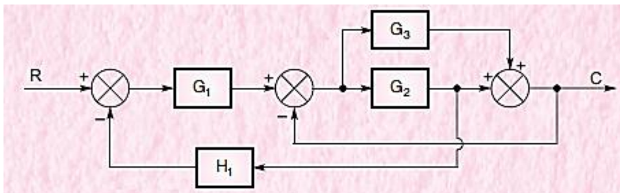
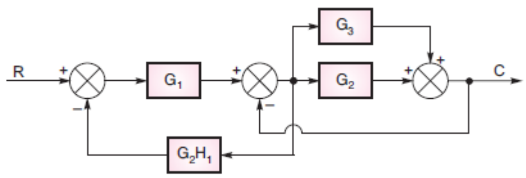


Fig. 1. Block diagram for example .

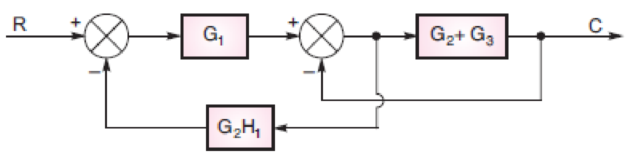
Solution:-

Step 1- Shift the take off point after block *G2* to a position before block *G2.*

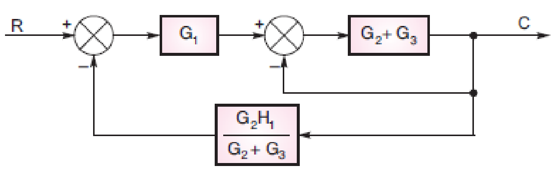


Block diagram reduction of system in Fig. .1.

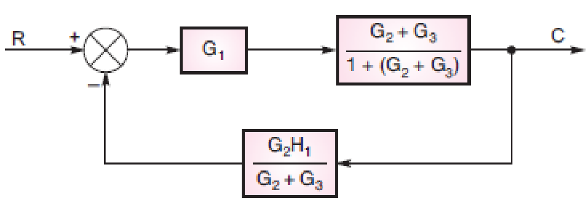
Step 2- Eliminate the summing point after block *G2*



Step 3- Shift the take off point before block (G2 + G3) to a position after block (G2 + G3).



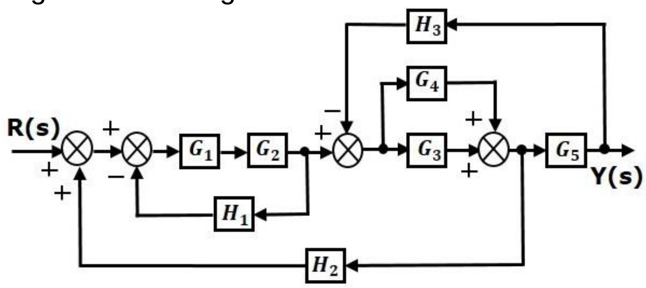
Step 4- Eliminate the summing point before block (G2 + G3).



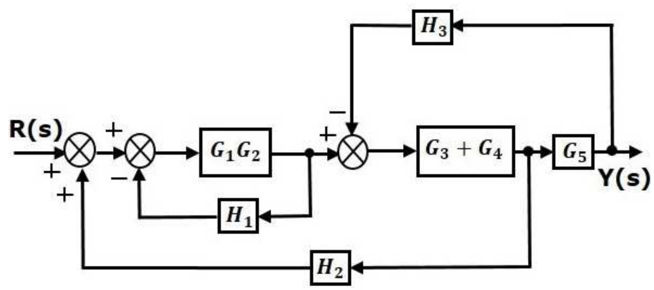
The overall transfer function determined below .

$$\frac{C}{R}=\frac{\dfrac{G_1(G_2+G_3)}{[1+(G_2+G_3)]}}{1+\dfrac{G_1(G_2+G_3)}{[1+(G_2+G_3)]}\cdot\dfrac{G_2H_1}{(G_2+G_3)}}=\frac{G_1G_2+G_1G_3}{1+G_2+G_3+G_1G_2H_1}$$

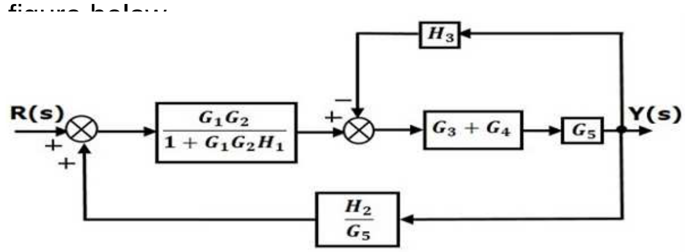
Example 2 - Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



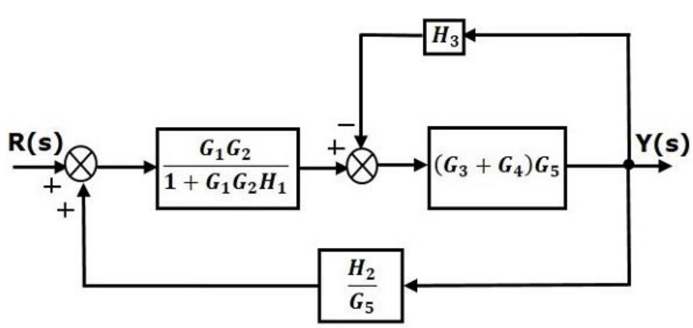
Step 1 – Use Rule 1 for blocks G1 and G2 .Use Rule 2 for block G3 and G4 .The modified block diagram is shown in figure below



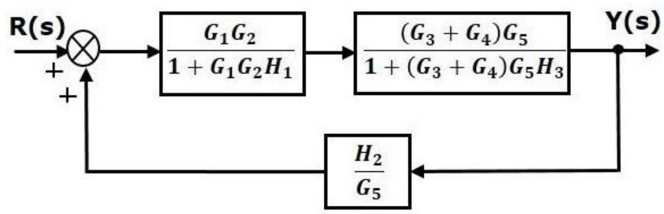
Step 2 – Use Rule 3 for blocks G1G2 and H1 .Use Rule 4 for shifting take-off point after the block G5.The modified block diagram is shown in the following figure below.



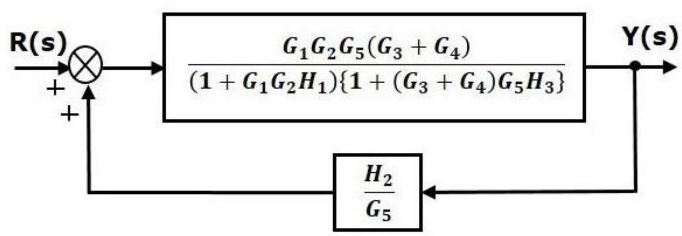
Step 3– Use Rule 1 for blocks (G3+G4) and G5 .The modified block diagram is shown in the following figure below .



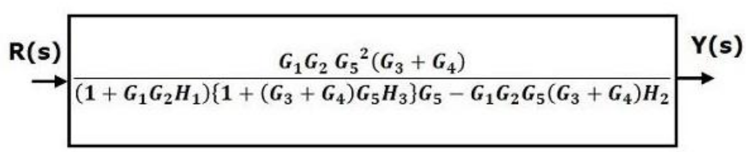
Step 4– Use Rule 3 for blocks (G3+G4) and H3 .The modified block diagram is shown in the following figure below .



Step 5 – Use Rule 1 for blocks connected in series .The modified block diagram is shown in the following figure below .



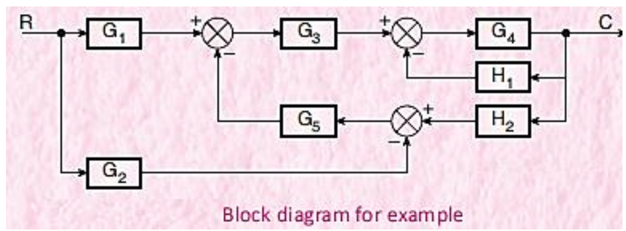
Step 6 – Use Rule 3 for blocks connected in feedback loop .The modified block diagram is shown in the following figure below .This is the simplified block diagram



Therefore the transfer function of the system is .

$$\frac{Y(s)}{R(s)}=\frac{G_1G_2G_5^2(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5-G_1G_2G_5(G_3+G_4)H_2}$$

Example 3 - Determine the overall transfer function for the block diagram

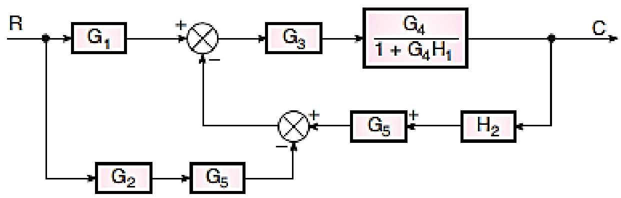


Block diagram for example

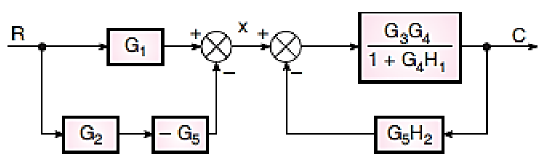
Solution:-

Step 1- Shift the summing placed before block G_5 towards left of block G_5.

Step 2- Eliminate the summing point located before block G_4. The block diagram takes the form Fig. below



Step 3- The summing points can be rearranged as shown in Fig. below .

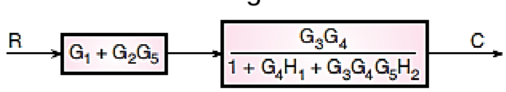


The blocks G_1 and G_2 ($-G_5$) are in parallel, the equivalence is $G_1 - G_2(-G_5) = G_1 + G_2G_5$

Step 4- Eliminating the summing point located before block ---- $G_3G_4/(1 + G_4H_1)$

$$\frac{C}{x} = \frac{\frac{G_3G_4}{1+G_4H_1}}{1+\frac{G_3G_4}{1+G_4H_1}\times G_5H_2} = \frac{G_3G_4}{1+G_4H_1+G_3G_4G_5H_2}$$

The block diagram takes the form as shown in Fig. below.

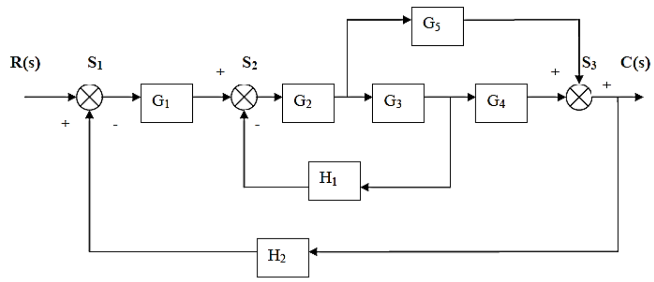


Step 5- The two blocks are in cascade,

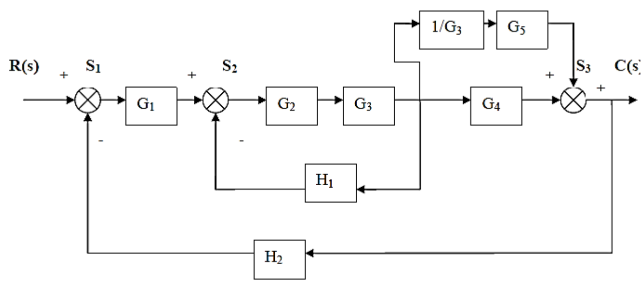
$$\therefore \quad \frac{C}{R} = (G_1 + G_2G_5)\times\frac{G_3G_4}{1+G_4H_1+G_3G_4G_5H_1}$$

$$\frac{C}{R} = \frac{G_1G_3G_4 + G_2G_3G_4G_5}{1+G_4H_1+G_3G_4G_5H_1}$$

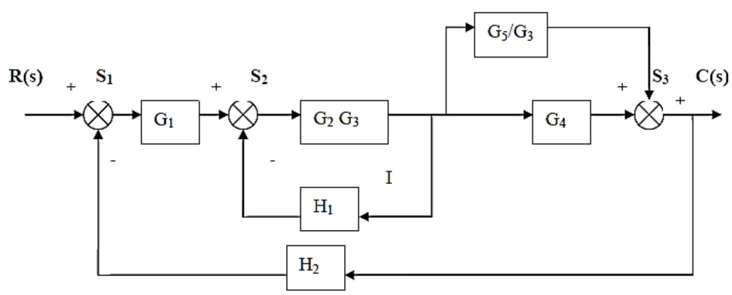
Example 4 - Reduce the block diagram shown in fig. below and obtain C(s) / R(s)



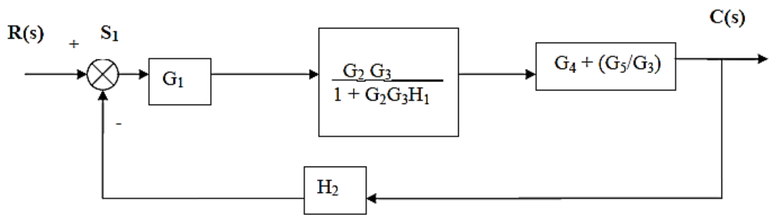
Step :1 Shifting the take off point between the blocks G2 and G3 to after the block G3



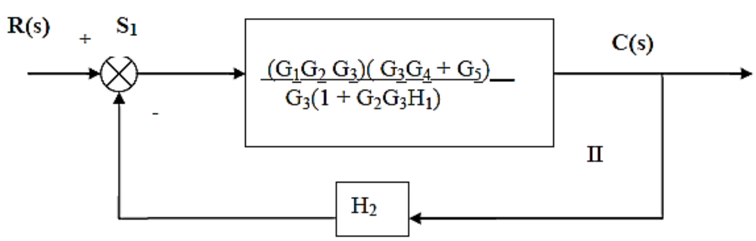
Step :2 Combine the blocks G2 and G3, 1/G3 and G5 which are in series.



Step :3 Eliminate feed back loop I, and then combine blocks G4 and G5 / G3 which are in parallel.



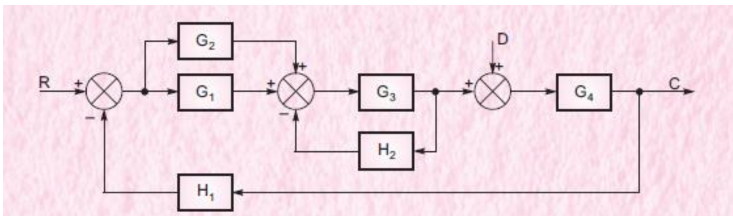
Step :4 Combine the blocks which are in series.



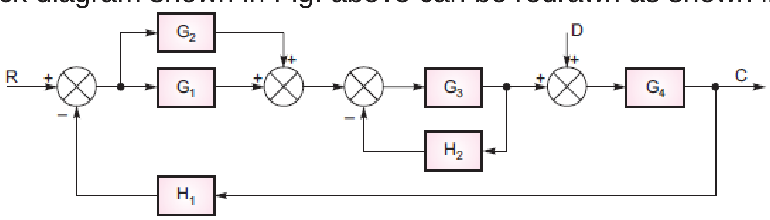
Step :5 Eliminate the feed back loop

$$\frac{C(s)}{R(s)} = \frac{\dfrac{(G_1G_2\,G_3)(\,G_3G_4 + G_5)}{G_3(1 + G_2G_3H_1)}}{1 + \left(\dfrac{G_1G_2G_3(G_3G_4 + G_5\,)}{G_3(1 + G_2G_3H_1)}\right) H_2} = \frac{G_1G_2G_3\,(G_3G_4 + G_5\,)}{G_3(1 + G_2G_3H_1) + [G_1G_2G_3\,(G_3G_4 + G_5)]\,H_2}$$

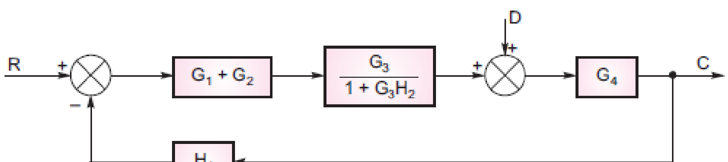
Example 5 - *Determine the ratio C(s)/R(s), C(s)/ D (s) and the total output for the system whose block diagram is shown in Fig. below.*



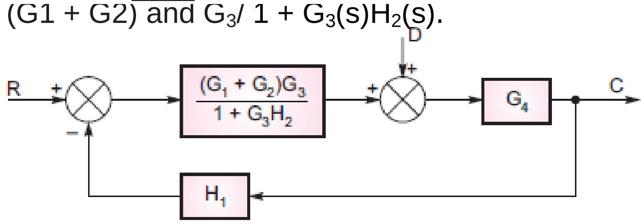
Solution. The block diagram shown in Fig. above can be redrawn as shown in Fig. below .



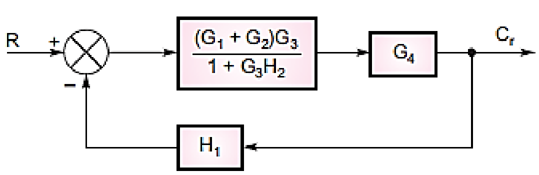
step 1- Eliminate summing points after block G1 and before block G3



step 2- Combine blocks (G1 + G2) and G_3/ 1 + G_3(s)H_2(s).



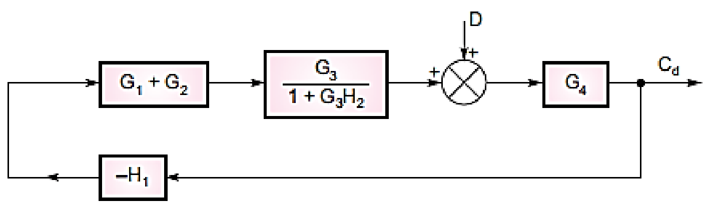
(i) In order to determine the ratio *C(s)/R (s)* Consider ***D*** = 0, the corresponding output is denoted as C_r.



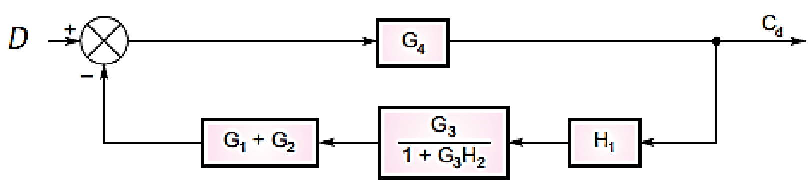
According to block diagram shown in above the ratio $C_r(s) / R(s)$ is determined below:

$$\frac{C_r}{R} = \frac{\frac{(G_1+G_2)G_3}{(1+G_3H_2)} \cdot G_4}{1+\frac{(G_1+G_2)\,G_3}{(1+G_3H_2)} \cdot G_4 \cdot H_1} = \frac{G_1G_3G_4 + G_2G_3G_4}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1}$$

(ii) Consider $\boldsymbol{R} = 0$, the corresponding output is denoted as C_d, hence the block diagram takes the form as shown below .



The block diagram shown in Fig. above is redrawn as shown in Fig. below.



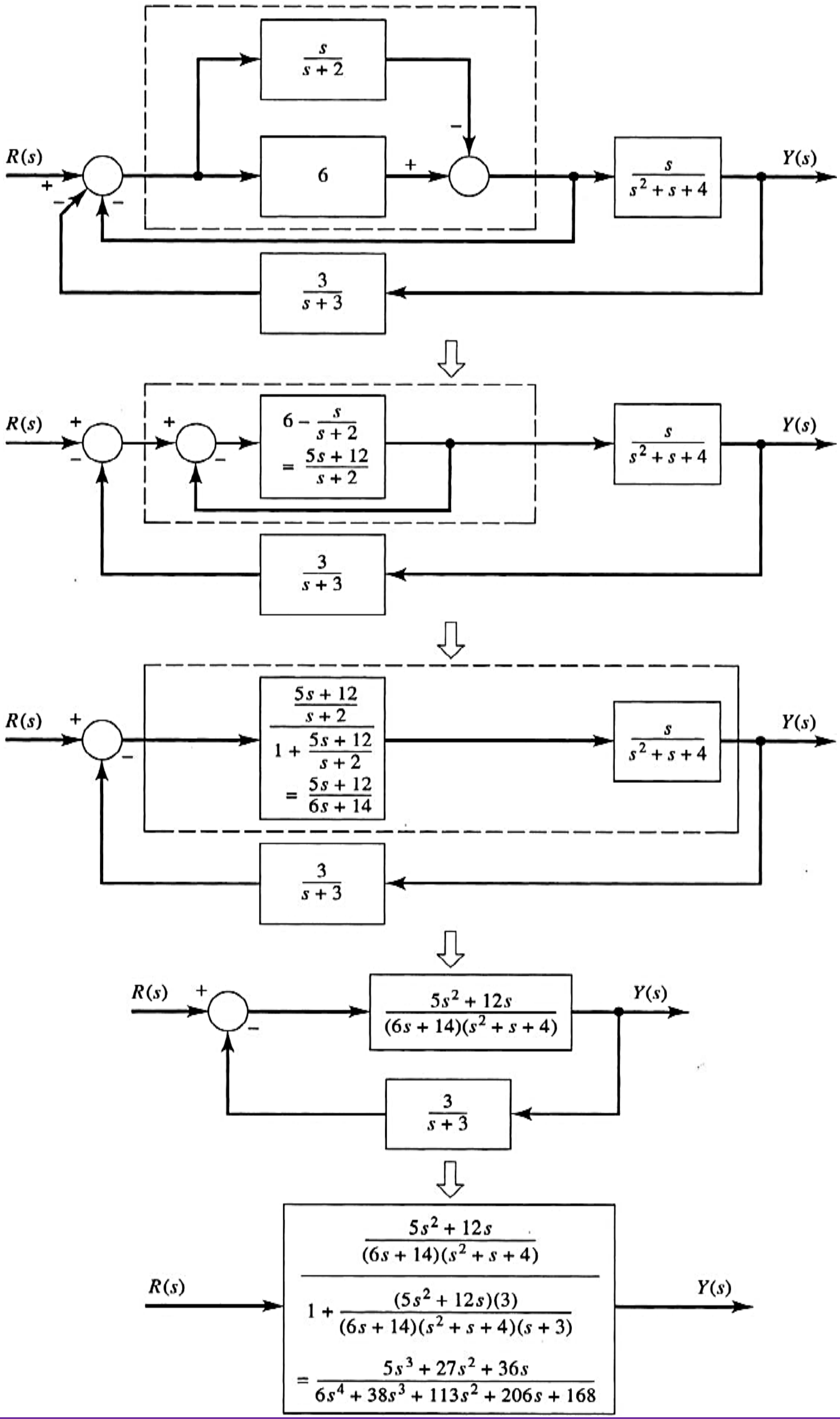
According to block diagram shown in Fig. above the ratio $C_d(s)/D(s)$ is determined below.

$$\frac{C_d}{D} = \frac{G_4}{1+G_4 \cdot \frac{(G_1+G_2)G_3H_1}{(1+G_3H_2)}} = \frac{G_4(1+G_3H_2)}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1}$$

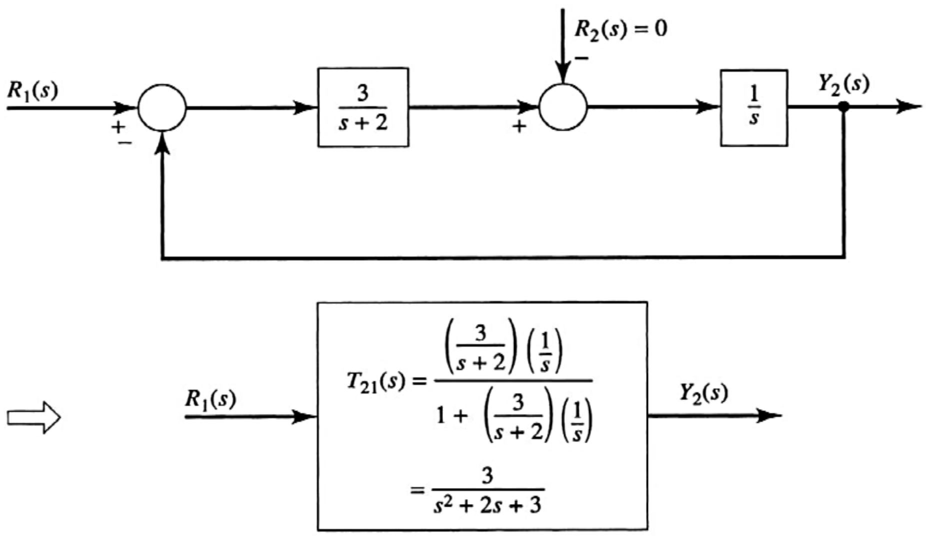
The total output is given by,

$$C = C_r + C_d = \frac{G_1G_3G_4+G_2G_3G_4}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1} \cdot R + \frac{G_4(1+G_3H_2)}{1+G_3H_2+G_1G_3G_4H_1+G_2G_3G_4H_1} \cdot D$$

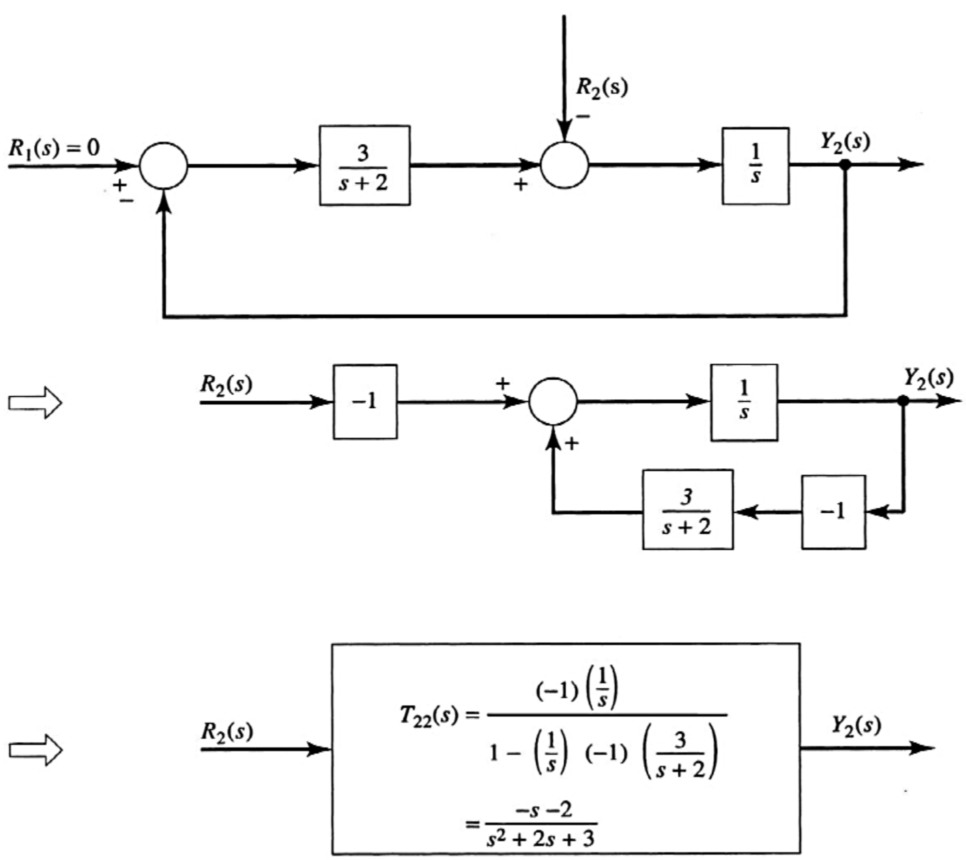
Example (6). Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



Example (7). Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



(*a*)



(*b*)